

Week 3 Seminar: Heuristics and Logic

Knights and knaves in logic, minimax by hand, and trusting a proof

Work in groups of 3 or 4. This sheet is **not assessed**. Activity 1 is a recap at the start of the session (5 minutes). Activities 2 and 3 follow the heuristics half of the lecture, and Activity 4 the logic half. Activities 5 and 6 practise last week's adversarial search by hand: draw the trees large, in pencil. Activity 7 is a 10-minute discussion. Finish anything you do not reach at home.

A Recap

Activity 1 | Remember Week 1?

5 minutes, start of session

On an island, every person is either a *knight* or a *knave*. Knights always tell the truth. Knaves always lie. A says: "We are both knaves." B says nothing.

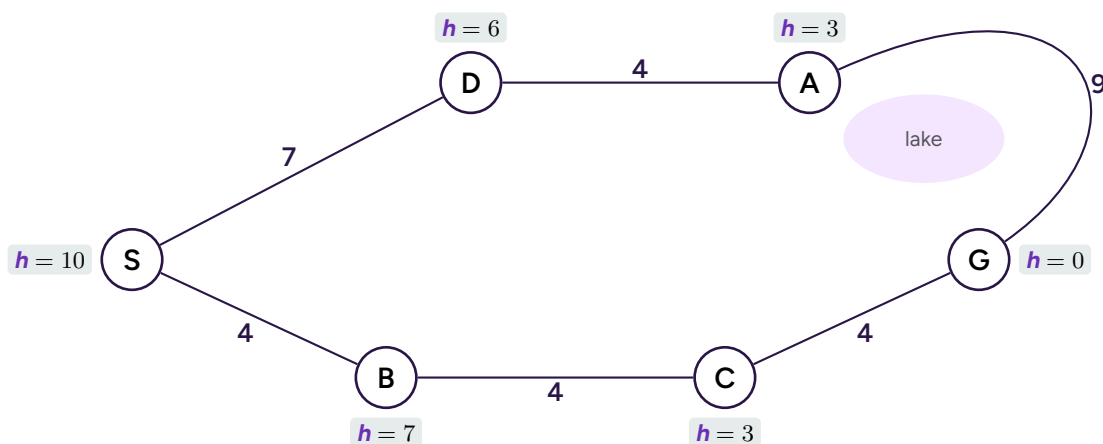
- (a) What are A and B?
- (b) How did you work it out in Week 1? Describe your method in words.

B Heuristics

Activity 2 | Greedy and A* by hand

15 minutes

A driver wants to get from S to G. The numbers on the roads are lengths in km. The number in each grey box is h , the straight-line distance from that town to G (rounded down), which never overestimates the real distance.



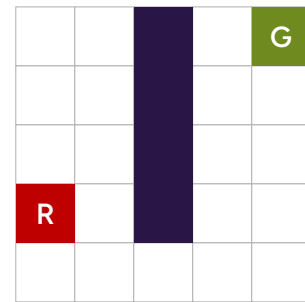
- Run **greedy best-first search** from S: always take the town in the frontier with the smallest h . In what order are the towns taken out? Which route does it find, and how long is it?
- Run **A*** from S: always take the town with the smallest $f = g + h$, where g is the distance driven so far. Fill in the table, one row per town taken out. Which route does A* find, and how long is it?
- Which towns did A* never take out of the frontier? Why did it not need to?
- A faulty map says $h = 20$ for C. Does this heuristic still never overestimate? Run A* again with $h(C) = 20$. Which route does it return now?

Step	Take out (f)	Add to the frontier: $g + h = f$	Frontier after
1	S ($f = 0 + 10 = 10$)		
2			
3			
4			
5			

Activity 3 | Is it a good guess?

15 minutes

A robot moves one square at a time (up, down, left or right) from R to G. Dark squares are walls. Here are five possible heuristics for a square:



1. the Manhattan distance to G (rows apart + columns apart);
 2. the straight-line distance to G, in squares;
 3. $h = 0$ for every square;
 4. twice the Manhattan distance;
 5. the number of columns apart only.
- (a) Work out each of the five heuristics for the square R. How many steps is the real shortest path from R to G?
- (b) Which of the five heuristics are *admissible*: they never overestimate, for any square? (Hint: why can walls only make the real path longer?)
- (c) Of the admissible ones, which would help A* most? Why?
- (d) What does A* do with heuristic 3? And what could go wrong with heuristic 4?

C Logic

Activity 4 | Knights and knaves, in logic

15 minutes

Week 1's harder puzzle. A says: "We are the same kind." B says: "We are of different kinds." Use four symbols: *AKnight*, *AKnave*, *BKnight* and *BKnave*.

- (a) Write A's statement, "we are the same kind", as a sentence in propositional logic. (Hint: both knights, or both knaves.)
- (b) Write B's statement, "we are of different kinds", in the same way.
- (c) Write four "if ... then" sentences: what follows if A is a knight, if A is a knave, if B is a knight, and if B is a knave?
- (d) The sentence "each person is a knight or a knave, but not both" leaves only four worlds. Check each one against your sentences, and fill in the table. Which world is left?
- (e) A new puzzle. A says: "At least one of us is a knave." B says nothing. Write the knowledge base, check the four worlds, and say what A and B are.

A is a	B is a	Fits the knowledge base? If not, which sentence does it break?
knight	knight	
knight	knave	
knave	knight	
knave	knave	

D Adversarial search by hand

Reminder from Week 2

In a game tree, **MAX** picks the move with the *highest* value and **MIN** picks the move with the *lowest*. Work from the bottom up: write a value in every node, then MAX's best move at the top is the branch with the highest value. With **alpha-beta pruning**, the search skips branches that cannot change the answer.

Activity 5 | Draw the tree, then solve it

15 minutes

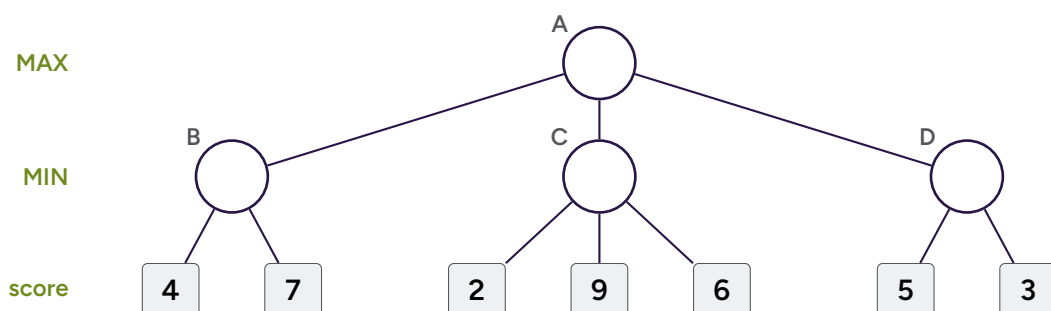
Four coins lie in a row: **5p, 20p, 10p, 2p**. Two players take turns. On each turn a player takes **one coin from either end** of the row. MAX goes first and keeps the coins she takes; MIN keeps the rest. MAX wants her total to be as large as possible, and MIN wants it to be as small as possible.

- (a) Draw the game tree. Write the row of coins that is left in each node, and label each branch with the coin taken. When one coin is left, MIN must take it, so stop there and write MAX's total.
- (b) Use minimax to find the value of every node. What should MAX take first, and how much will she get?
- (c) A *greedy* player always takes the larger of the two end coins. What would MAX get by playing greedily on the first move, against a perfect MIN? What does this show?

Activity 6 | Alpha-beta pruning

10 minutes

MAX moves at A, B, C and D are MIN nodes. Search the tree from left to right.



- (a) Without pruning, what is the minimax value of B, C, D and A? Which move does MAX make?
- (b) Now search with alpha-beta. After B, MAX knows it can get at least 4. The first leaf under C is 2. Which leaves under C never need to be looked at? Explain why in one sentence.
- (c) Does pruning change MAX's move, or the value of A?
- (d) Suppose the leaves under C were in the order 9, 6, 2 instead. Would anything under C be pruned? What does this tell you about the order in which moves are searched?

E Discussion

Activity 7 | Discuss: can we trust a proof?

10 minutes

A railway signalling system has been checked by model checking: in every world that fits its knowledge base, no two trains are ever sent onto the same stretch of track.

- (a) Can we now trust the system completely? What could still go wrong?
- (b) Who should write the knowledge base, and who should check it?
- (c) What should happen when the railway changes, for example when a new line opens?

Portfolio link

Logic gives answers that are certain and easy to explain, but only as good as the knowledge written into it. When you judge in Part 2 whether a rule-based method fits a problem, use Activity 7: who writes the rules, who checks them, and what happens when the world changes?